PISTON THEORY APPLIED TO WING-BODY CONFIGURATIONS: A REVIEW OF THE MATHEMATICAL BASIS

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Abstract: Piston theory [1--3] has long been used in obtaining the aerodynamic loading on flight-vehicle surfaces at supersonic and hypersonic speeds. The appeal of piston theory to the aeroelastician is that surface pressures are related by a point-function relationship to the local surface motion and inclination; this renders the method very computationally inexpensive. In planar flows, nonlinear piston theory offers the further advantage over classical linear theory of accounting for thickness and camber effects [4]; moreover, it may be applied at higher Mach numbers.

Research devoted to the investigation of flutter of wings, shells, and panels has included work towards extending piston theory to improve the accuracy of loading predictions [4]. The similarity of piston theory to other approximate analytical methods with different physical bases was briefly reviewed by Meijer and Dala [5]. One significant extension to piston theory -- local piston theory (LPT) -- was developed by Zhang et al [6]: piston theory was applied as a small perturbation about a mean steady flow computed by a higher fidelity aerodynamic method. This was exploited to reduce the cost associated with unsteady computational fluid dynamics (CFD); the accuracy of the loading prediction at reduced cost fueled a widespread application of LPT, and the method is now typically associated with application to CFD. Piston theory has thus received renewed interest in literature.

Significantly, LPT has increasingly been applied to flight vehicles, as opposed to isolated aerodynamic surfaces. One such recent study [7] noted decreased accuracy of load prediction compared to the usually accurate results obtained for isolated surfaces. Whilst the theoretical basis for piston theory for non-interfering flows has been well-examined, its applicability to flows where interference is significant has not received attention. This has motivated the present paper on investigating the mathematical basis for applying piston theory to wing-body configurations.

The present paper investigates the theoretical basis for applying piston theory to wing-body configurations from consideration of the governing (Euler) equations of the flow. Particular attention is given to the mathematical development of piston theory by Il'yushin [3] and to Sychev's [8] method in investigating the conditions under which a point-function relationship for surface pressures may be derived. The perturbation form of the governing equations is also considered to investigate the application of local piston theory and as a preliminary consideration towards applying local piston theory to flows with viscosity.

Keywords. Aeroelasticity, Piston Theory, Interference.

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