DEVELOPMENT OF THE AIRCRAFT FLIGHT PATH OPTIMIZATION PROGRAMM

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Abstract. To have a non-stationary flight conditions on altitude, velocity, angle of attack, g-load, etc. is typical for the aircrafts with the rocket engine. Pure aerodynamical criteria appear to be insufficient while choosing parameters of the aircraft aerodynamical layout. In addition flight properties of the aircraft strongly depend on its flying path. Thereupon aircraft flight path optimization program was developed. This program solves the variation problem by the means of the direct numerical method – polynomial method. It utilizes a genetic method to solve a multi-variable function extremum searching problem. As its first application, the task of the wing surface optimization (criterion – minimal flight time) was solved. The optimization shows that it is better to have no wing on small ranges and for a long flight range it is better to have a large wing surface for the aircrafts with the rocket engine to achieve minimum flight time.

Keywords. Optimization, flight path, direct method.

1 Introduction

This article is devoted to the direct flight path optimization method description and its realization by the means of the computer program. The need to have a fast and robust flight path optimization method occurs for the problems concerned with the aircrafts which have a considerable non-stationary flight conditions. The main idea of the direct method is to approximate aircraft flight path by the sequence of polynomials to move from the variation problem to the multi variable function optimization problem. To solve the latter one can utilize the range of numerical methods each of which has both advantages and disadvantages. The number of tests showed that the family of gradient methods is not well consistent with the task considered because of discontinuous efficiency function and problems with gradient calculation. Therefore the genetic method was chosen for the optimization problem which also allows one to introduce efficiency function limitations in a rather simple way [6].

The method described was applied practically in the task of wing surface optimization for the aircraft with the rocket engine. The problem was to choose the aircraft layout parameters having its aerodynamical parameters and engine behavior to provide the minimal flight time on the fixed range.

2 Theoretical base

2.1 Optimization method selection

Direct numerical methods which can be easily applied for the tasks with lots of limitations on both phase and control variables are widely spread in the flight dynamic problems. Direct methods allow us to move from the variation problem [1-3] to multivariable function extremum searching problem. The latter task can be solved via the range of numerical methods: Newton method, the set of gradient methods, penalty function method, genetic method, etc. But those of methods which demand gradient calculation [4] have one common disadvantage: requirement for the efficiency function to be continuous. Moreover the optimization problem with limitations requires the additional means introduction for the gradient methods family.

The increase in the last ten years of the modern computers calculation capability provides the resources for the optimization methods based on the direct searching of the efficiency function extremum. Being saved from the problems with efficiency function discontinuity direct searching methods can be successfully applied within the range of function optimization problems. It is the genetic method [6] that was chosen for the multivariable function optimization problem because of the following advantages:

1. There is no need to calculate any of functions derivatives;
2. Function should not be defined within the whole range of parameters variation;
3. It is quite simple to include limitations on both phase and control variables.

2.2 Direct numerical method – polynomial method

In this section we briefly introduce the idea of the polynomial method. Let us consider the $I[\boldsymbol{x}(t)]$ functional minimization problem, knowing that $inf\ (I[\boldsymbol{x}(t)]) = m > -\infty$. Assume that we found the sequence of vector-functions satisfying settled boundary conditions, and:

$$lim_{n\to\infty} I[\boldsymbol{x}_n(t)] = m.$$

Generally the sequence of the vector-functions $\boldsymbol{x_n(t)}$ appears to converge to the vector-function $\boldsymbol{x(t)}$ and $\boldsymbol{I}[\boldsymbol{x}(\boldsymbol{t})] = \boldsymbol{m}$. Let us consider n-parametric functions family, satisfying the boundary conditions:

$$\boldsymbol{x}(n, t_0) = \boldsymbol{a_0}\,, \boldsymbol{x}(n, t_k) = \boldsymbol{a_k}. \tag{2.1}$$

Functions family $\boldsymbol{x}(n,t)$ is designed as follows. All the range $t_0 < t < t_k$ is divided by the points t_i $(i = 1 \div n)$ on $(n+1)$ undefined segments satisfying the next condition:

$$\min_i(t_i - t_{i-1}) \to 0, \text{when } n \to \infty, (i = 1 \div n), t_{n+1} = t_k.$$

The values of t_i $(i = 1 \div n)$ are the family parameters. Within the each segment $t_{i-1} < t < t_i$ $(i = 1 \div n + 1)$ function $\boldsymbol{x}(n,t)$ is defined as

$$\boldsymbol{x}(n,t) = \boldsymbol{f_i}(t - t_i), \tag{2.2}$$

where $\boldsymbol{f}_i(t) = \boldsymbol{\varphi_{i0}} + \sum_{j=1}^{p} C_{ij}\, \boldsymbol{\varphi_j}(t)$; $\boldsymbol{\varphi_j}(t) -$ coordinate functions.

Using functions (2.2) one can treat functional $I[\boldsymbol{x}(n,t)]$ as the function of coefficients C_{ij} and parameter t_i.

$$I[x(n,t)] = \Phi\big(C_{ij};t_i\big), (i = 1 \div n+1, j = 1 \div p). \tag{2.3}$$

Some part of the coefficients C_{ij} is defined from the boundary conditions (2.1), another part – from the function $x(n,t)$ continuity requirement and possibly $x(n,t)$ derivatives continuity requirement at the ends of the segments. The rest of the coefficients C_{ij} is defined from the optimization problem solution for the function $\Phi\big(C_{ij};t_i\big)$.

Numerical investigations into the atmospheric flight dynamics problems show, that provided the proper choice of coordinate functions, one can obtain rather good method convergence with the small values of n ($n = 0$, 1 or 2 depending on the particular boundary conditions).

One of the convenient methods for the flight path $\boldsymbol{\widetilde{H}} = \boldsymbol{\tilde{f}}(\boldsymbol{L})$ parameterization is polynomial method. All the range $\boldsymbol{L_0} \le \boldsymbol{L} \le \boldsymbol{L_k}$ is divided into the segments $L_{i-1} \le L \le L_i$, $i = 0 \div k$. Within the i-th segment function $\boldsymbol{\widetilde{H}_i} = \boldsymbol{\tilde{f}_i}(\boldsymbol{L})$ is defined as the 5th degree polynomial:

$$\begin{gathered}\widetilde{H}_i = c_{0i} + c_{1i}(L - L_{i-1}) + c_{2i}(L - L_{i-1})^2 + c_{3i}(L - L_{i-1})^3 + c_{4i}(L - L_{i-1})^4 \\ + c_{5i}(L - L_{i-1})^5, \;\; i = 1 \div n.\end{gathered} \tag{2.4}$$

Using the 5th degree polynomial one is able to independently vary the next parameters at the i-th point:

- coordinates $\widetilde{H}_i(L_i), i = 1 \div n$;
- derivatives $\widetilde{H}_i{}'(L_i), i = 1 \div n$;
- second derivatives $\widetilde{H}_{i_-}^{\,''}(L_i)$ and $\widetilde{H}_{i_+}^{\,''}(L_i), i = 0 \div n+1$;

Applying equations for the flight path boundary conditions and for the both $\boldsymbol{\widetilde{H}} = \boldsymbol{\tilde{f}}(\boldsymbol{L})$ and its first derivative continuity condition one will have the following equations system for some of the coefficients $\boldsymbol{c_{ji}}$ definition:

$$\begin{cases} c_{0i} = \widetilde{H}_{i-1}, \\ c_{1i} = \widetilde{H}'_{i-1}, \\ 2c_{2i} = \widetilde{H}''_{+\,i-1} \\ \displaystyle\sum_{j=0}^{5} c_{ji}(L_i - L_{i-1})^j = \widetilde{H}_i, \\ \displaystyle\sum_{j=1}^{5} jc_{ji}(L_i - L_{i-1})^{j-1} = \widetilde{H}'_i, \\ \displaystyle\sum_{j=2}^{5} j(j-1)c_{ji}(L_i - L_{i-1})^{j-2} = \widetilde{H}''_{-\,i}; \end{cases} \tag{2.5}$$

$$i = 1 \div n+1.$$

2.3 Physical model

Physical model includes the system of equations for the mass point flat movement under the gravitation force, aerodynamical force and engine propulsion:

$$\begin{cases} \frac{dV}{dt} = g(n_{xa} - sin\theta), \\ \frac{d\theta}{dt} = \frac{g}{V}\left(n_{ya} - cos\theta\right), \\ \frac{dL}{dt} = Vcos\theta, \\ \frac{dH}{dt} = Vsin\theta, \\ \frac{dG}{dt} = -\frac{P}{J}, \\ n_{xa} = \frac{Pcos\alpha - X}{G} = \frac{Pcos\alpha - C_{xa}qS}{G}, \\ n_{ya} = \frac{Psin\alpha + Y}{G} = \frac{Psin\alpha + C_{ya}qS}{G}; \end{cases} \tag{2.6}$$

Where

- $C_{xa}(\alpha, M, H)$, $C_{ya}(\alpha, M)$ – aerodynamical drag and lift coefficients, respectively;
- q – ram air flow;
- L, H, V – flight distance, altitude and speed;
- θ – pitch;
- P, J – engine propulsion, specific pulse;
- ρ – air density at altitude H;
- M, a – Mach number, sonic speed;
- S – reference surface;
- G – aircraft weight;

To carry out the integration of the movement equation system, one has to define the operation for the aircraft along the flight path $\widetilde{H} = \tilde{f}(L)$.

$$n_{ya} = \frac{V^2}{\mathrm{g}} cos^3\theta[\tilde{f}''(L)] + cos\theta. \tag{2.7}$$

For the aircraft to have the defined g-load is necessary to find the corresponding angle of attack:

$$\frac{Psin\alpha + C_{ya}(\alpha)qS}{\mathrm{G}} - n_{ya} = 0. \tag{2.8}$$

While solving the equation (2.8) one can bring into action the angle of attack limitation:

$$\alpha_{min} < \alpha < \alpha_{max}. \tag{2.9}$$

The following method was applied to synthesize the operating algorithm [5]. Assume that the process of aircraft return on the reference path corresponds to the desired differential equation with constant coefficients:

$$\frac{d^2\Delta H}{dt^2} + 2\xi\Omega\frac{d\Delta H}{dt} + \Omega^2\Delta H = 0, \tag{2.10}$$

where $\Delta H = H - \widetilde{H}$ is the difference between the real altitude H and the altitude of the reference path $\widetilde{H}$, $\boldsymbol{\xi}$ and $\boldsymbol{\Omega}$ is the constant coefficients. Using (2.4) and (2.6) one can obtain:

$$n_{xa}(\alpha)\,sin\big(\theta - \tilde{\theta}\big) + n_{ya}(\alpha)cos\big(\theta - \tilde{\theta}\big) =$$
$$= \cos\big(\tilde{\theta}\big)\,[1 + \frac{1}{g}\tilde{f}''(L)\left(\frac{dL}{dt}\right)^2 - 2\frac{\xi\Omega}{g}(Vsin\theta - \tilde{f}'(L)\frac{dL}{dt} - \frac{\Omega^2}{g}\big(H - \widetilde{H}\big). \tag{2.11}$$

The equation (2.11) defines the g-load value directed normally to the reference path for the current value of L. The (2.11) equation is similar with (2.7) when $H = \widetilde{H}$ and $\theta = \tilde{\theta}$.

2.4 Program review

Developed the aircraft flight path optimization program including the following main features:

- Optimization of the aircraft flying path via the direct numerical method: the polynomial method. Optimization criterion: minimal flight time;
- Genetic method for the multivariable function extremum searching;
- Numerical integration of the movement equation system. Aircraft is considered as a mass point moving in the vertical plane under aerodynamic forces, gravitation and propulsion. In addition, while integrating, aerodynamics and engine behaviour is taken into account.

3 Investigation results

The program developed was applied practically in the optimal wing surface searching problem when the flight range is fixed and minimum flight time is required. The numerical research was carried out for the number of different flight ranges and wing surface values. The following source data were provided: aerodynamical coefficients (three variants for the different wing surface values), engine behaviour and limitations on the angle of attack, normal g-load, flight time and ram air flow.

Research carried shows the following influence of the wing surface on the average horizontal speed (calculated as the flight distance divided by the flight time) when the distance is fixed and path is optimal:

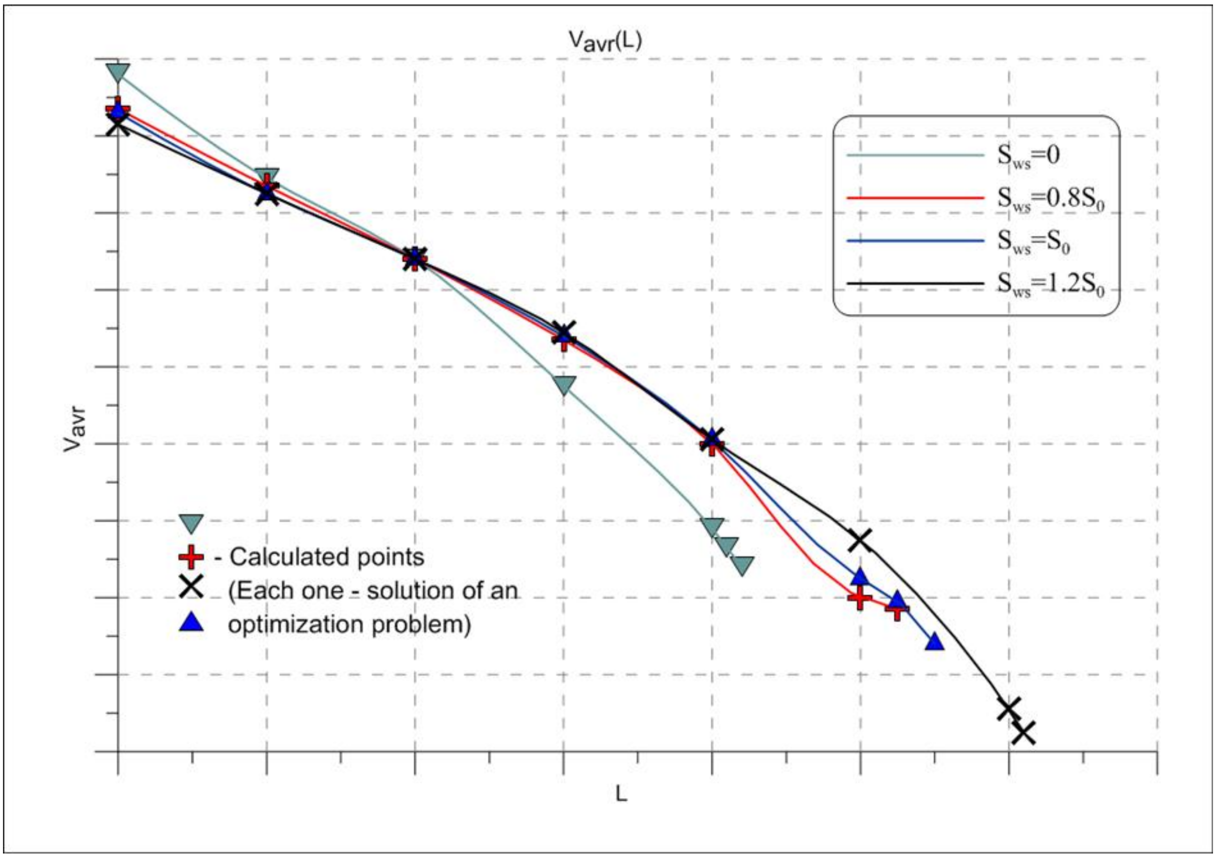

Figure 1: The average flight speed on the optimal flight path.

The numerical research shows that due to the aerodynamical features of the aircraft considered (the main part of the drag coefficient is provided by the aircraft forebody) it is better to have no wing for the minimal flight time on the small ranges and to have the bigger wing surface on the long flight ranges. In addition the bigger wing surface allows aircraft to achieve the larger ranges.

While carrying out the optimization genetic algorithm shows rather good efficiency: with 1 polynomial for path approximation (2 independent variables), population of 250 individuals and 10 optimization steps algorithm converges in no more than 10 minutes:

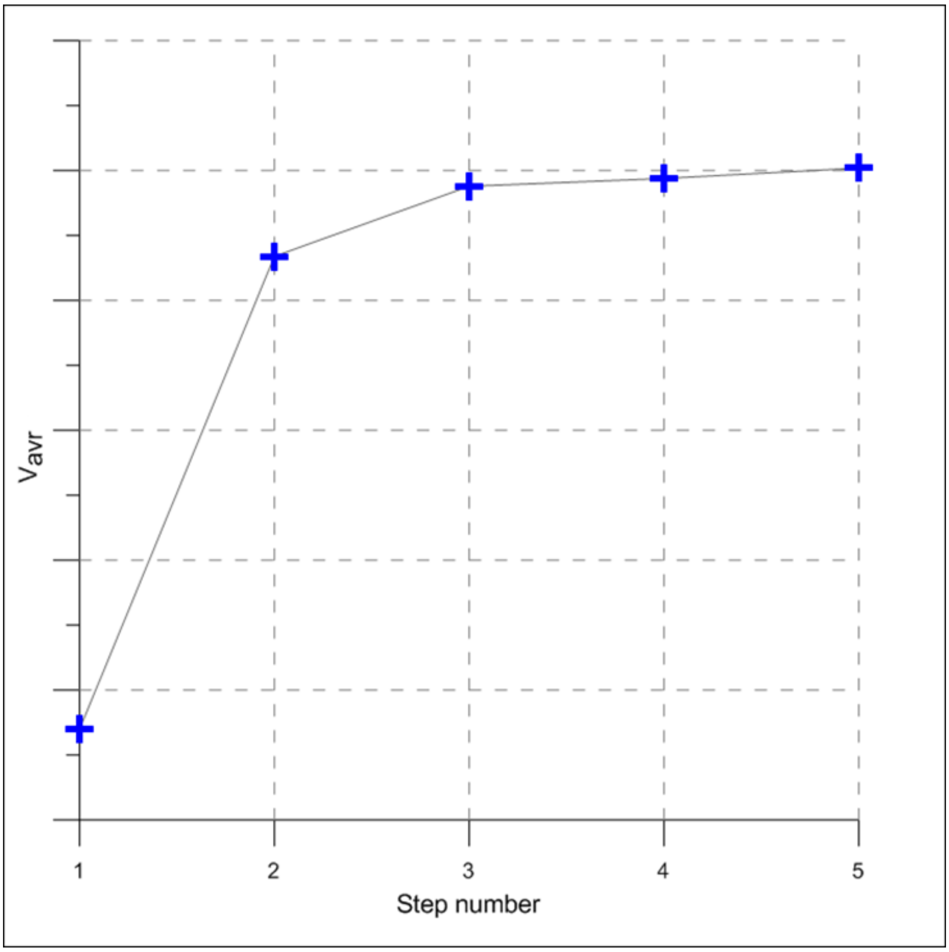

Figure 2: Genetic algorithm in action. Fitness function of the best individual on each step.

Conclusion

The program was developed and applied for the investigation into the optimal wing surface choosing problem. This program makes it possible to find the optimal flight path for the given distance while the path is approximated by the sequence of 5^{th} degree polynomial. The optimization criterion – average flight speed calculated as the flight distance divided by the flight time;

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