Solution of the inverse problem for an airfoil within the framework of the Reynolds averaged Navier – Stokes equations

A.L. Bolsunovsky	N.P. Buzoverya	I.A. Gubanova	M.A. Gubanova
TsAGI	TsAGI	MIPT	TsAGI
		irinka_gubanova@mail.ru	

Abstract. A new iterative method for solving the inverse problem for an airfoil in a compressible gas flow within the framework of RANS equations has been developed. The method belongs to the class of residual correction principle methods. Following to this principle an airfoil geometry correction is defined from the residual between computed and desired pressure distributions. The inverse problem is being solved iteratively by means of consequent calls of the direct analysis method and the correction block.

The program package ANSYS CFX is used as a tool for the direct method. Correction block is based upon the previously developed domestic inverse full-potential code. The correction block's processing time is negligible in comparison with the run time of direct method.

The proposed algorithm provides fast convergence and high accuracy of the results. Examples of solving the inverse problems for an airfoil in subsonic and transonic flows are given. These results show high efficiency of the method. The algorithm could be developed for solving more complicated inverse problems, including two-dimensional ones for multielement airfoil and three-dimensional problems.

Keywords. Inverse problem, Navier – Stokes equations, ANSYS CFX, airfoil, pressure distribution.

Introduction

There are two families of aerodynamic methods. The first family concerns direct analysis methods. With these methods the properties of a given aerodynamic shape are computed. The shape is optimized either by means of trial and error or, more and more often nowadays, by optimization techniques. The second family consists of the inverse methods. With these methods a certain shape is computed based on required properties. Typically, pressure distribution is specified and an inverse method seeks for the airfoil shape which produces it.

Inverse methods enable to remove or weaken shocks, reduce flow disturbances at the specified location, to realize the pressure distribution favorable for the development of a laminar or turbulent boundary layer etc. Not every specified pressure distribution can be realized physically as the inverse problem is incorrect in the general case. The solvability conditions are precisely defined only for two-dimensional potential flows [1-3], and for the real flows - viscous, transonic, and particularly three-dimensional one must use engineering approaches.

One of the most common approaches to the construction of the inverse method is based on the principle of residual correction, according to which the deformation of the airfoil shape is determined from the residual between the computed and target (Cp*) pressure distributions [4-13]. The problem is solved iteratively by means of consequent calls of the direct method and the correction block (Fig.1). Naturally, the more reliable is the geometry correction, the smaller is the required number of iterations for convergence.

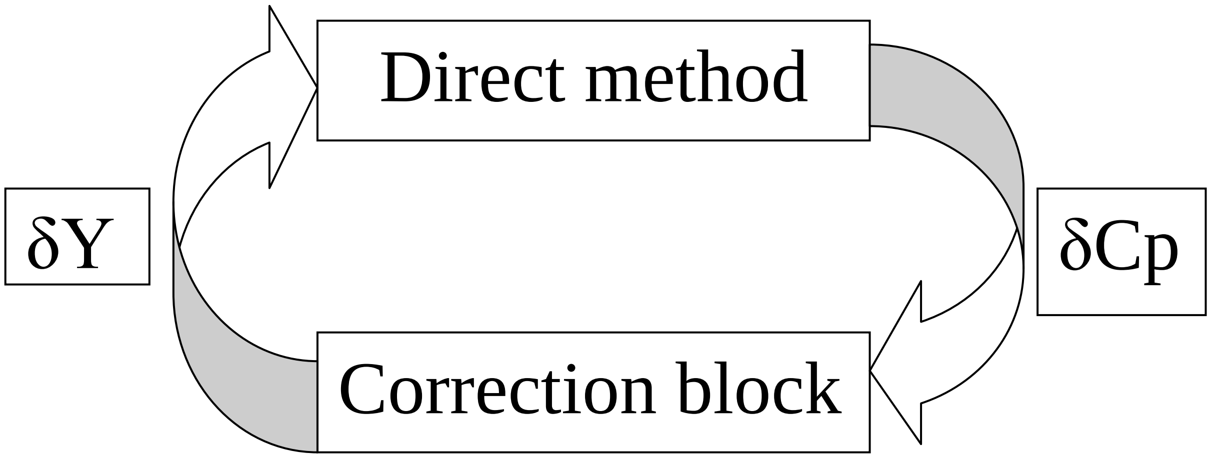

Figure 1: Residual correction method

In the residual correction approach well-developed direct flow solvers are used as a "black box" and can be easily replaced later by more sophisticated methods. The development of direct methods is usually ahead of the development of purely inverse methods, so the principle of residual correction enables getting the solution of an inverse problem for a wider class of flows and conditions.

Recently the rapid development of computational aerodynamics led to the widespread practical application of Navier-Stokes solvers [14-19]. These methods represent a rational compromise between the complexity of the equations and required computational and time resources. Using of more simple equations leads to significant acceleration of the calculations, but it is not such universal and requires careful analysis because of the risk to miss some important physical property of the flow.

An inverse two-dimensional method within the framework of the RANS equations proposed in this paper also belongs to the class of residual correction methods. The main idea consists in the fact that as a corrector that generates current modifications of the airfoil contour the inverse full-potential method is used. This method - TRAINV – has been developed in TsAGI many years ago [12] and is widely used for the design of high-speed isolated airfoils, and as a corrector in 3-dimensional method for solution the inverse problem for transonic wing [13].

1. Statement of the problem and a description of the computational algorithm

In this paper the inverse problem of finding the coordinates of the airfoil contour for a given pressure distribution is under consideration. The method used belongs to the class of residual correction methods. The principle of nested levels, according to which the geometry correction block is precisely the method of inverse problem of a lower level, is used to create an efficient algorithm. Here level means complexity of the equations or the geometry. Typical levels of geometry in aerodynamics are airfoil, wing, wing + fuselage, complete layout. Typical levels of equations complexity are Laplace equation, full potential equation, Euler equations, Navier-Stokes equations. Using the principle of nested levels one can gradually increase the complexity of inverse problem in hand, while maintaining a clear understanding of the physics of the phenomena, and rationally distribute calculations at different levels to minimize computational efforts. For example, the method developed in TsAGI to solve the inverse problem for a wing in transonic flow - TRAWDES [13] consists of three levels (Fig.2).

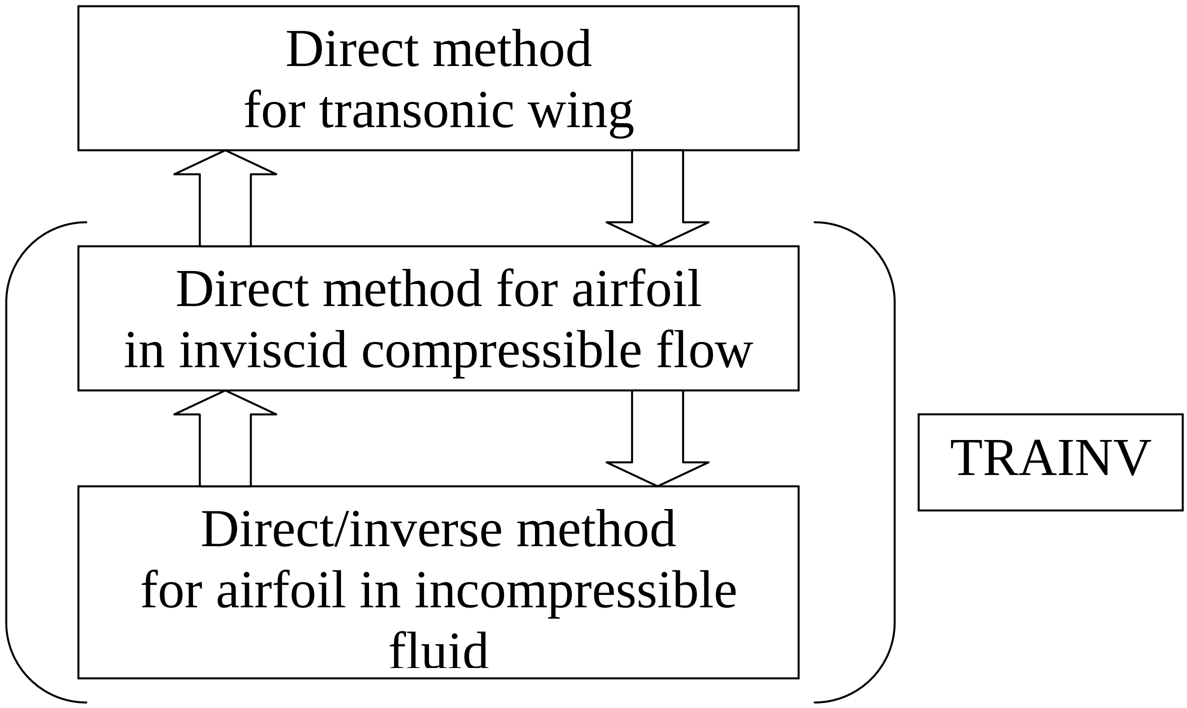

Figure 2: The principle of nested levels

An iterative inverse method proposed in this paper also consists of three levels (Fig.3). The upper level includes a method for the direct calculation of the airfoil in a viscous compressible flow - a software package ANSYS CFX [20]. The second level is the direct calculation of the airfoil in the inviscid compressible flow by full potential method. The first (lowest) level is a method for solving the direct / inverse problem for an airfoil in incompressible fluid. The first and the second levels in the aggregate constitute a method for solution the inverse problem for the airfoil in the inviscid compressible flow - code TRAINV [12].

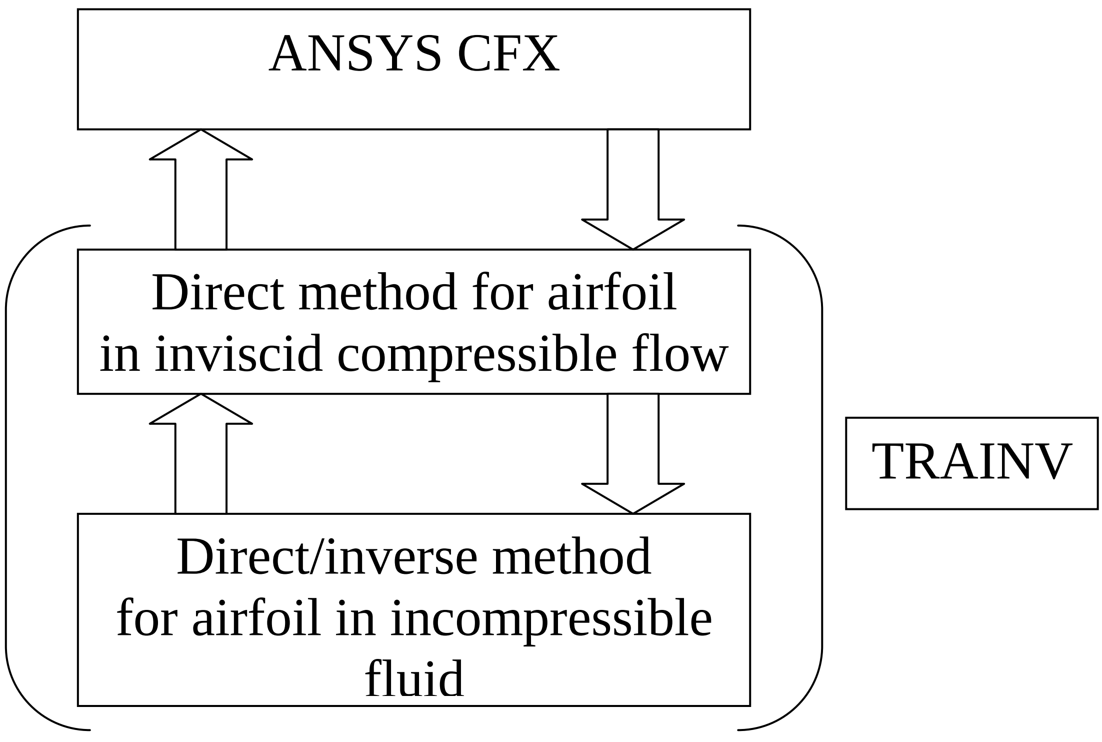

Figure 3: Applying the principle of nested levels in the given problem

The inverse problem is solved for the airfoil in the process of iterations. The following actions are fulfilled at each k-th iteration:

1. Calculation of the airfoil which geometry has been obtained at the previous (k-1)-th iteration by CFX and determination of the residual δCpk as the difference between the current and the target pressure distribution.
2. Calculation of the same airfoil by full potential method.
3. Forming the target pressure distribution for the airfoil in the inviscid compressible flow by adding the difference r • δCpk (where $r \leq 1$) to the pressure distribution from step 2.
4. Solving the inverse problem for airfoil in the inviscid compressible flow with target pressure distribution from step 3 and obtaining (k +1)-th airfoil geometry.

At each iteration, the pressure residual $\varepsilon_{cp}=\sqrt{\frac{1}{N}\sum_{i}^{N}(Cp_i - Cp_i^*)^2}$ is calculated. The process is repeated several times until convergence or the reaching the minimum achievable residual. Sufficient for practice convergence corresponds to $\varepsilon_{cp} \leq 0.01$.

Thus, solving the inverse problem for the proposed algorithm requires a serial call of different software packages. For this purpose the authors have written a script - the batch file in the Python programming language.

2. A brief description of the calculation methods

2.1 Direct method for compressible viscous flow

ANSYS CFX software package is designed to solve a wide range of problems of Fluid Dynamics, chemical kinetics, combustion, multiphase flow dynamics and others [20]. For the finite-difference approximation of the Reynolds averaged Navier – Stokes equations finite volume method is used. It is possible to carry out calculations on multi-block structured and unstructured and hybrid meshes as well. In this work, the calculations were performed on the multi-block structured meshes, allowing obtaining good resolution of the viscous boundary layer and wake.

The calculations were performed on a grid of "C" type with approximately 100000 cells. The grid was constructed by ICEM CFD. When changing the airfoil shape in the process of iteration the initial mesh is automatically tuned to the corresponding geometry. The height of the first cell is $\Delta y \sim 10^{-6}$ (parameter y+ near the wall is from 0.3 to 0.6). The boundaries of the computational domain are removed from the airfoil to 50 chords. Two-parameter differential model SST is selected as a turbulence model. The flow was supposed to be fully turbulent.

2.2 Direct full-potential method

For the calculation of an airfoil in the ideal transonic flow the program based on the mixed multi-grid method for solving the full-potential equation is chosen. It includes a fast method for solving the Poisson equation for a subsonic part of the flow and the method of successive linear over-relaxation for the supersonic part. The solution is carried out on a grid of "O" type in the transformed plane of variables, obtained by successive transformations from airfoil exterior to the interior of the unit circle. The calculation is carried out successively on 3 grids: 32 × 8, 64 × 15 and 128 × 15 cells. The calculation of the airfoil at single angle of attack requires about one-tenth of a second on a modern PC.

2.3 Direct / inverse method for incompressible flow

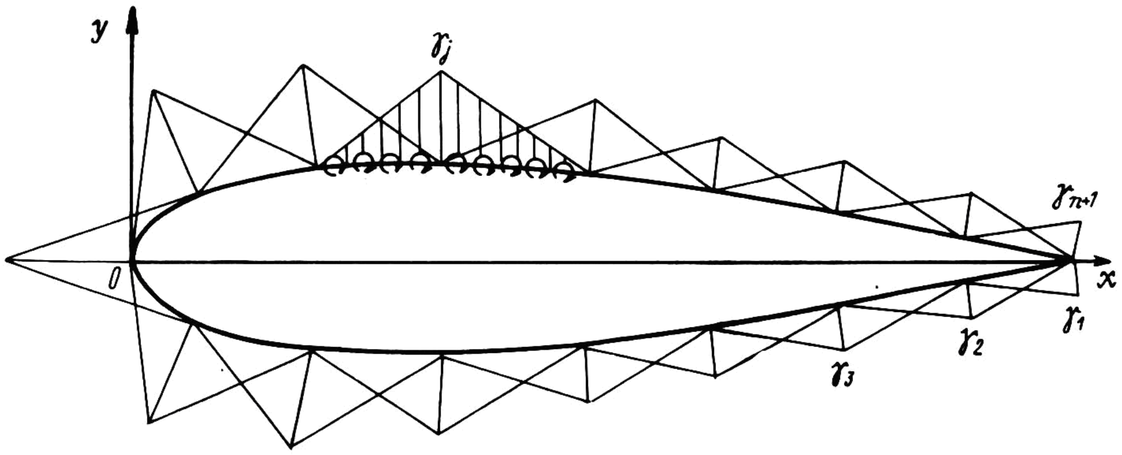

Figure 4: Partition of airfoil contour on the panel

For the calculation of an airfoil in an incompressible fluid the panel method with piecewise linear distribution of the vorticity on the surface is applied (Fig.4). Direct and inverse problems are solved with identical boundary conditions and vorticity distributions. The condition of the stream function constancy in the airfoil contour nodes

$$\psi_i = y_i Cos\alpha - x_i Sin\alpha - \frac{1}{2\pi}\oint \gamma(s) ln\sqrt{[x_i - x(s)]^2 + [y_i - y(s)]^2}\, ds = const$$

i=1, 2, …, N+1

results in the direct mode in a system of linear algebraic equations with regard to unknown vortex intensities γ_j:

$$\sum_{j=1}^{N}(a_{ij} - a_{N+1j})\gamma_j = (x_i - x_{N+1}) Sin\alpha - (y_i - y_{N+1}) Cos\alpha$$

i=1, 2, …, N

In the inverse mode the integral equation can be solved directly relative to the contour coordinates:

$$y_i = \frac{1}{Cos\alpha}\left[\psi_c + \frac{1}{2\pi}\oint_C \gamma(s) ln\sqrt{(x_i - x(s))^2 + (y_i - y(s))^2}\, ds\right] + x_i tg\alpha \qquad (1)$$

or in a panel presentation

$$y_i = \frac{1}{Cos\alpha}\left[\psi_c - \sum_{j=1}^{N} a_{ij}\gamma_j\right] + x_i tg\alpha$$

i = 1,2,…,N+1

This favorably distinguishes the approach using the method of the stream function from the other known methods, which usually determine slopes instead of coordinates.

Equation (1) is non-linear, as the unknown values y_i are under the integral. The solution is sought in the process of iterations, until a new form of the airfoil is determined. At each of the iterations airfoil coordinates are transformed to canonical form and the angle of attack value is corrected. In case of “fish-tailing” or excessive disclosure of the trailing edge wedge addition to the ordinates of the airfoil is performed, which brings blunting of the trailing edge to the specified magnitude. Usually the convergence process needs from 10 to 30 iterations. Since the solution of the inverse problem for an incompressible fluid is only a sub-step of the common problem, it is not necessary to achieve complete convergence.

2.4 The solution of the inverse problem in a compressible potential flow

The solution of inverse problem for an airfoil in a compressible flow is based on the principle of residual correction, where the method described in section 2.2 is applied as the direct method, and as the method described in section 2.3 is applied as a corrector.

In case of pure subcritical flow pressure residual from the upper level is transmitted directly to the bottom, may be with some relaxation. After solving the inverse problem in an incompressible fluid the resulting airfoil geometry is smoothed in the region of critical point and transmitted to the upper level for the direct calculation. For transonic flows the iterative process described above needs to be modified. Determination of corrections to the airfoil geometry in the supersonic region should represent the hyperbolic character of the flow equations.

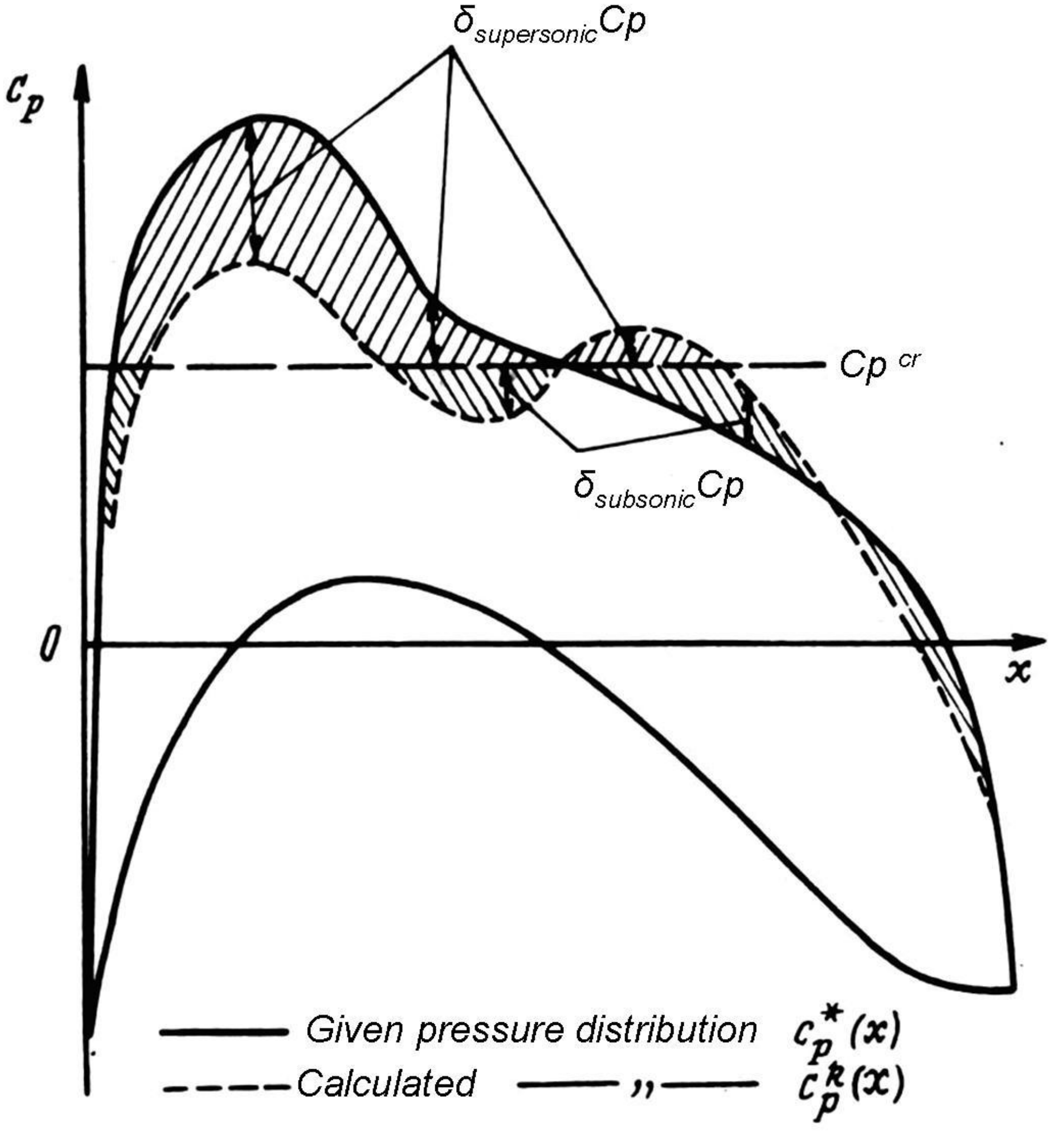

Figure 5: Splitting the difference between the calculated and target pressure distribution on the subsonic and supersonic parts

In general, the difference between the calculated and the target pressure distribution splits into subsonic and supersonic parts [6] (Fig.5):

$$\delta\, Cp^k = \delta_{subsonic}\, Cp^k + \delta_{supersonic}\, Cp^k$$

Supersonic part $\delta_{\text{supersonic}} Cp^k$ is used to obtain the modification of the surface slopes in the supersonic region according to Akkerett formula

$$\delta y' = 0{,}5\sqrt{M^2 - 1} \cdot \delta_{\text{supersonic}} Cp$$

Then the corrections of surface slopes in the supersonic area are converted to an equivalent subsonic distribution of pressure amendments $\delta_{\text{subsonic}} Cp^{eq}$. To this end two successive calculations of airfoil in subsonic flow are carried out by panel method: the first one with the initial surface slopes and the second one with modified surface slopes. General pressure correction in an incompressible fluid is defined as the sum of two components:

$$\delta \tilde{C} p^k = r\delta_{\text{subsonic}} Cp + q\delta_{\text{subsonic}} Cp^{eq} \quad , r \le 1,\ q \le 1$$

Further, the procedure is similar to the previous one, but requires an additional enhanced smoothing of airfoil geometry in the shock region.

3. Examples of solutions

The geometry recovery tests are often used to validate the inverse methods. In such tests target pressure distribution is taken from the direct calculation of the existing airfoil, and the initial geometry is arbitrary. These tests allow monitoring the convergence of the inverse method not only on the distribution of pressure, but also on the geometry. Note that for the general inverse problem the required geometry is unknown, and, indeed, there is no evidence of its existence.

Figure 6 shows an example of the inverse problem for a target pressure distribution obtained by fully turbulent RANS calculation of NACA 4412 airfoil at pure subsonic condition M = 0.2, α = 6°, Re = 4 • 10^6. Symmetric NACA 0010 airfoil and the angle of attack α = 3° are taken as the initial approximation. The figure shows the results obtained after five global iterations. Obtained airfoil shape and calculated pressure distribution are almost identical to the target ones. Since the corrector well describes the physics of the flow, it yields good result even at the first iteration. The figure also shows plots of the residuals and the angle of attack value.

In the following example (Fig.7) the inverse problem is solved for a target pressure distribution, obtained by calculation of the NACA 4412 airfoil at transonic condition M = 0.67, α = 2°, where shock exists on the upper surface. Iterations start from the NACA 0010 airfoil at the initial angle of attack α = 1°. Twelve iterations were required to solve the design problem. Fairly good agreement of the pressure distribution is obtained on the seventh iteration, but five additional iterations were necessary to receive good convergence on the geometry.

In the third example the target pressure distribution was chosen arbitrarily on the upper surface (Fig.8) to implement shock-free airfoil at M = 0.67. The pressure distribution on the bottom surface was not changed. As the initial approximation NACA 4412 airfoil was taken. Solution with acceptable accuracy has been obtained after ten iterations.

Conclusions

The algorithm and a program for solution of the inverse problem for an airfoil within the framework of the RANS equations have been developed. The solving is carried out iteratively by means of the residual correction procedure. Software package ANSYS CFX is used as a direct solver. The inverse two-dimensional full-potential method – TRAINV, developed earlier in TsAGI, is used as a correction block.

Given examples of inverse problem solutions in subsonic and transonic flow demonstrate the high efficiency of the method. The algorithm could be developed for solution of more complicated inverse problems, including two-dimensional ones for multielement airfoil and three-dimensional problems.

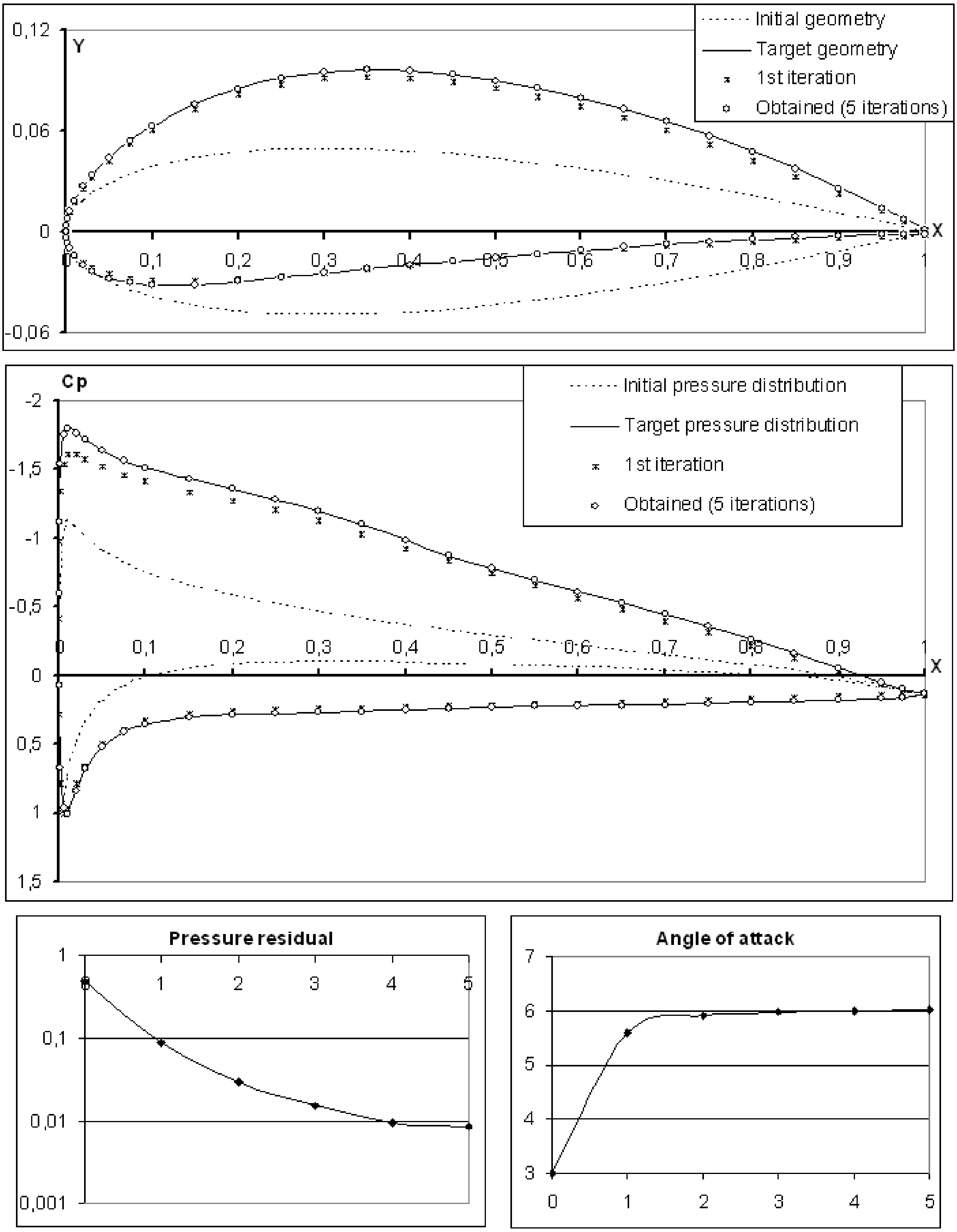

Figure 6: Inverse problem for subsonic flow

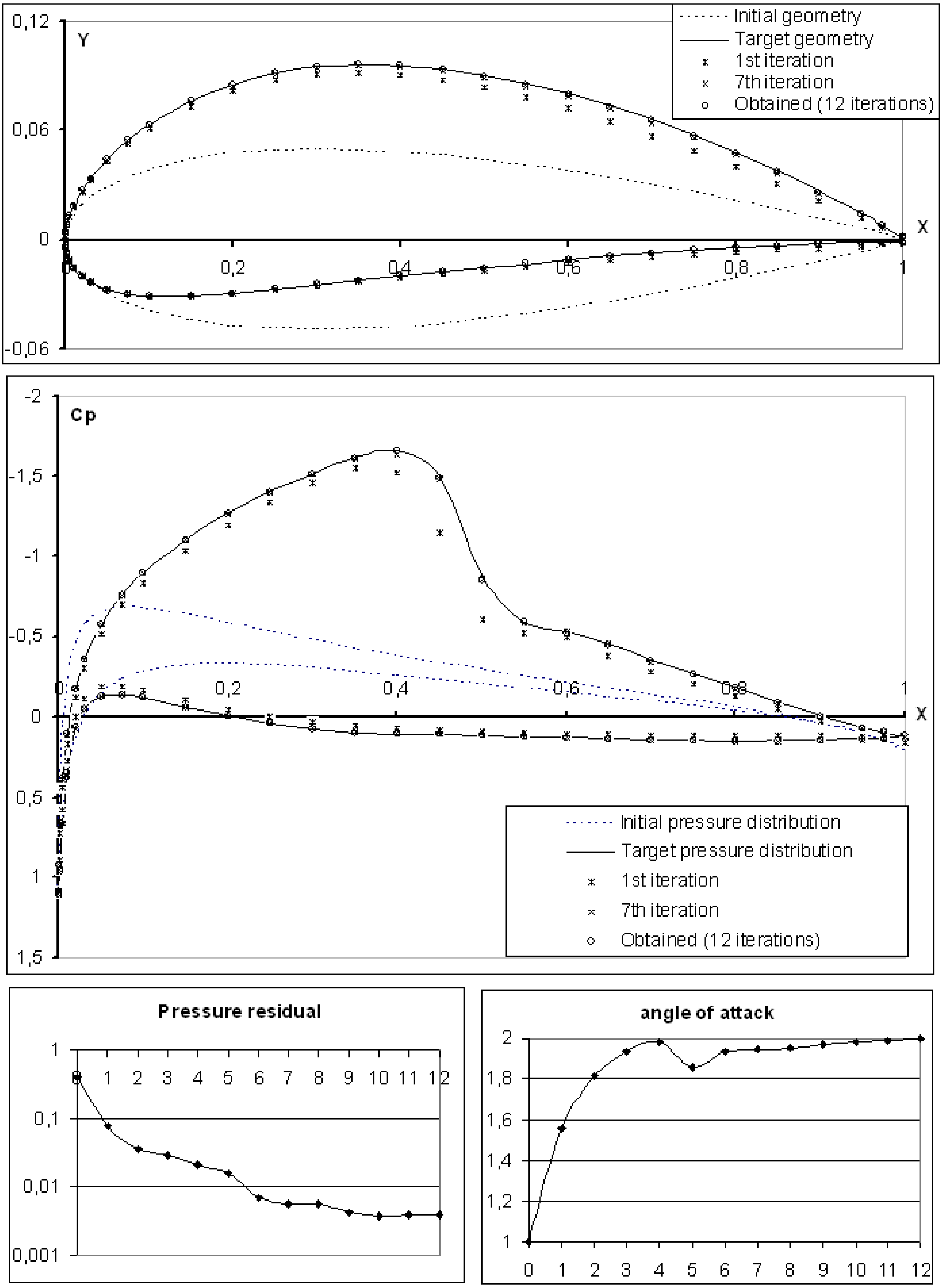

Figure 7: Inverse problem for transonic flow

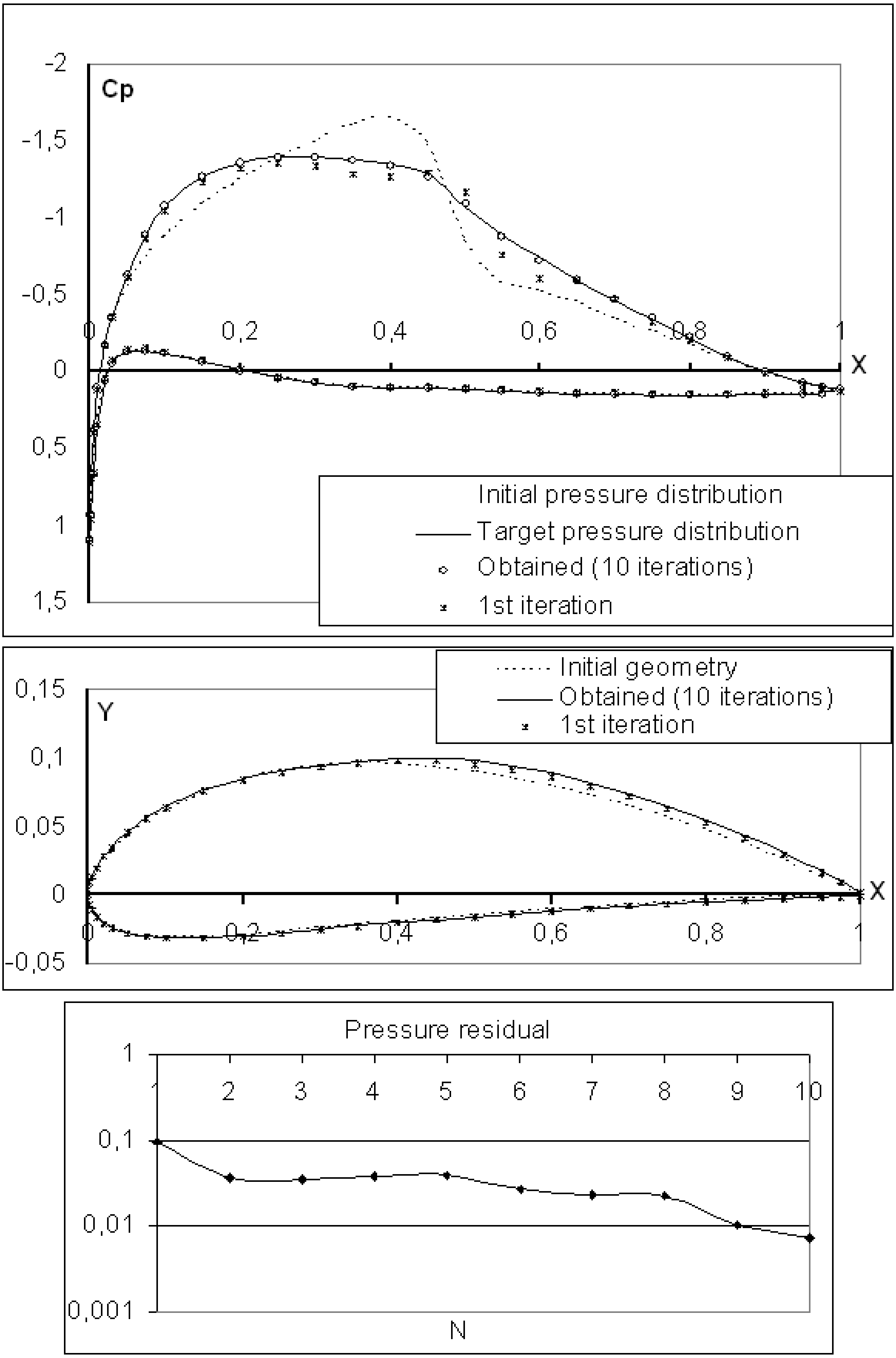

Figure 8: Inverse problem for the arbitrary target pressure distribution.

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